The Properties of

Model Rocket Body Tube Transitions

Date: September 6, 2000

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NAR 78048
CAR S680

Rev: 2 (June 12, 2005)

1.0 Introduction

When designing model rockets, designers often choose to incorporate different diameter body tubes into the models' design. Sometimes this is done to accommodate special features, such as payload or camera bays, but often it is done just to make the design more interesting and appealing. Component manufacturers usually offer standard transition parts that permit standard body tubes of different diameters to be mated, resolving the problem simply and conveniently. But designers and builders are usually left to their own devices when the mating problem involves body tube diameters that are not supported by the manufacturers, or perhaps require a non-standard transition length. Fortunately, the problem of the non-standard transition can be solved with a little geometry, and this paper presents some solutions.

1.1 Background

Geometrically, body tube transitions are a physical derivative of a cone. If one can imagine a cone that has some portion of its' top cut off, then the remainder of the cone would represent a transition body, gradually reducing in diameter from its' base to the point where the top of the cone was cut off, as shown in Figure 1, below:

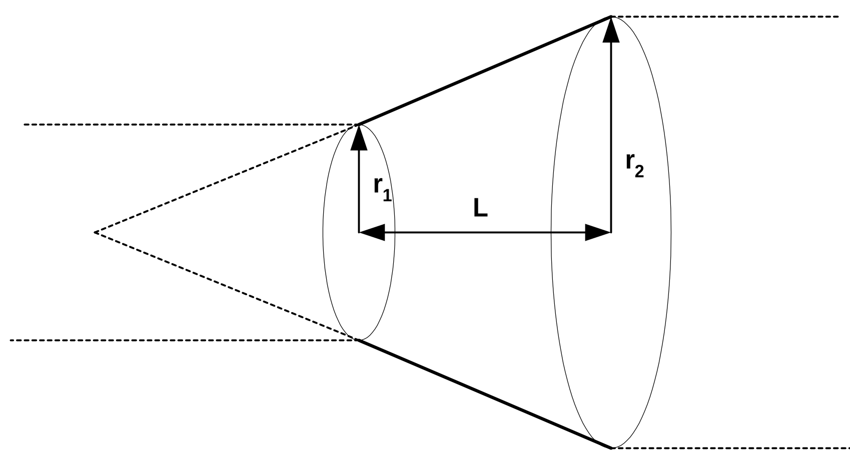

Figure 1: Transition Body

This transition body can be used to mate two different body tubes of a model rocket when the diameters of the base and top of the transition match the diameters of the two body tubes. The sharpness or steepness of the transition is proportional to the length **L** of the transition: the longer the length of the transition the more gradual the reduction in diameter.

Once the diameters of the body tubes that are to be mated are known, and the desired length of the transition is decided, it should be possible to create a solid transition out of a lightweight material such as balsa. Creating a solid transition is relatively straightforward if access to the right tools is available or if one is willing to spend the money to have one made by a component manufacturer, e.g.: BMS. But most often, a solid part isn't necessary. The most common practice is to create a transition shroud, which when constructed, has the same shape and serves the same purpose as the solid transition but is nothing more than a conically-derived covering of the junction between the two coupled body tubes.

The next section defines and explains the geometry behind transition shrouds and then derives the basic equations necessary to calculate the parameters of the part.

2.0 Transition Shrouds

2.1 Transition Shroud Geometry

Taking the decision to make a transition shroud means that the builder has designed some other direct means to solidly couple the two body tubes. This is usually achieved with some sort of coupling tube/centering ring arrangement. The main point is that the builder is not relying on the transition shroud to provide strength to the airframe: instead, the transition shroud is being used to fair the joint between the two dissimilar tubes and will only be strong enough to protect itself. Usually, the shroud is made from paper or card stock (or even light gauge plastic sheet or fiberglass) depending on the size of the model. Sometimes underlying reinforcement ribs made of balsa are installed to support and further strengthen the shroud.

The first step involved in creating a transition shroud consists of defining the dimensions the shroud will require. These values are then plugged into a set of relations that define the layout dimensions of the shroud. But before we do this, we must first establish the relationship that exists between the shroud's three-dimensional shape and its plan-form layout. It is the plan-form layout that we must determine in order to fashion it correctly from an essentially two-dimensional material such as paper, cardboard or plastic cardstock.

Let's revisit the geometric properties of the transition:

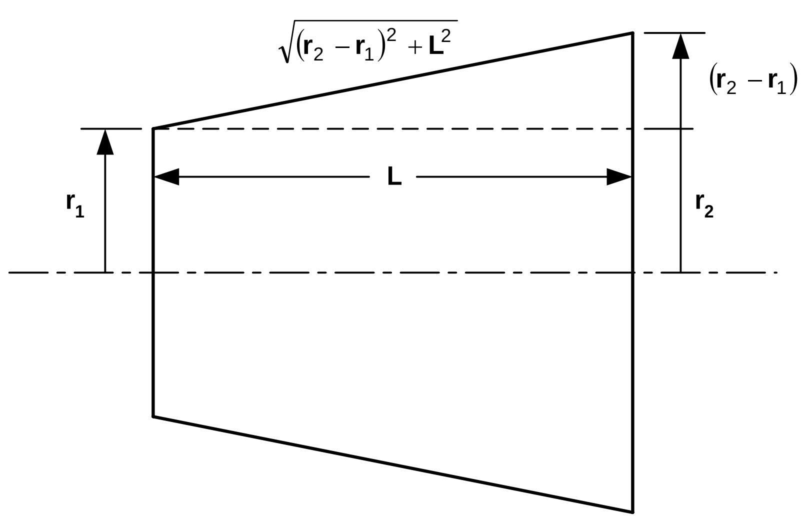

Figure 2: Transition Side View

In Figure 2, the transition is shown in the side view. In this shape the transition shroud is defined by:

- **L,** the length of the transition,

- $\mathbf{r_1}$, the radius of the front of the transition,
- $\mathbf{r_2}$, the radius of the rear of the transition.

These dimensions are known to the designer, as the body tube diameters and the desired length of the transition are parameters that the designer specifies for the rocket design.

It is also apparent that $\mathbf{r_1}$ and $\mathbf{r_2}$ correspond to the radii of the smaller and larger body tubes respectively. By using the Pythagorean Theorem, the shroud length can be defined as $\sqrt{(\mathbf{r}_2 - \mathbf{r}_1)^2 + \mathbf{L}^2}$.

If one were to take a pair of scissors and cut along the axis of the shroud, the transition could be laid flat on a table. In doing so, the shroud would assume the shape of a disc sector, as shown in Figure 3 below:

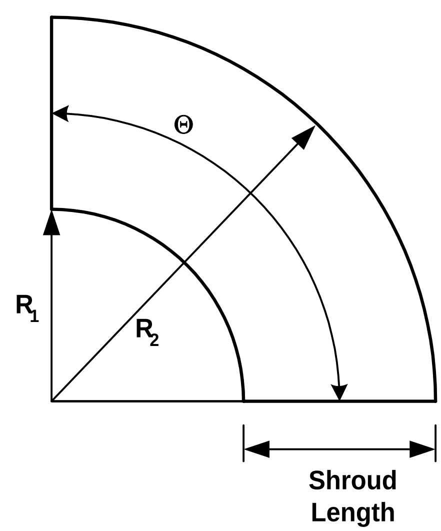

Figure 3: The Transition Shroud in Plan Form.

The physical characteristics of the sector are defined by:

- $\mathbf{R_1}$, the radius of the inner circle,
- $\mathbf{R_2}$, the radius of the outer circle,
- Θ°, the sector length in degrees.

These are the dimensional properties that must be computed in order to correctly fabricate the shroud.

2.2 Solving for R_1 and R_2

Let's take a closer look at these properties.

The arc lengths of the sector can be defined as:

- $S_1 = \Theta^r R_1$, with Θ expressed in radians.
- $S_2 = \Theta^r R_2$

Since Θ is common to both arc lengths,

$$\therefore \qquad \frac{\mathbf{S}_1}{\mathbf{R}_1} = \frac{\mathbf{S}_2}{\mathbf{R}_2}$$

Because the sector is directly derived from the shroud, it follows that the plan form properties are directly related to the transition shroud properties. We can begin to define some of these relationships as follows:

- $S_1 = 2\pi r_1$ and $S_2 = 2\pi r_2$

  (By definition, the arc lengths must be equal to the circumferences of the respective ends of the transition).

$$\therefore \qquad \frac{2\pi\mathbf{r}_1}{\mathbf{R}_1} = \frac{2\pi\mathbf{r}_2}{\mathbf{R}_2};$$

$$\frac{\mathbf{r}_1}{\mathbf{r}_2} = \frac{\mathbf{R}_1}{\mathbf{R}_2};$$

$$\therefore \qquad \mathbf{R}_1 = \frac{\mathbf{r}_1\mathbf{R}_2}{\mathbf{r}_2}$$

$$\text{and} \qquad \mathbf{R}_2 = \frac{\mathbf{r}_2\mathbf{R}_1}{\mathbf{r}_1}$$

By comparing the diagrams in Figures 2 and 3, it can be seen that the shroud length $\sqrt{(\mathbf{r}_2 - \mathbf{r}_1)^2 + \mathbf{L}^2} = (\mathbf{R}_2 - \mathbf{R}_1)$.

$$\mathbf{R}_2 = \mathbf{R}_1 + \sqrt{(\mathbf{r}_2 - \mathbf{r}_1)^2 + \mathbf{L}^2}\ .$$

Substituting the above for $\mathbf{R}_2$ gives:

$$\mathbf{R}_1 = \frac{\mathbf{r}_1}{\mathbf{r}_2}\left[\mathbf{R}_1 + \sqrt{(\mathbf{r}_2 - \mathbf{r}_1)^2 + \mathbf{L}^2}\right]$$

$$= \frac{\mathbf{r}_1\mathbf{R}_1}{\mathbf{r}_2} + \frac{\mathbf{r}_1}{\mathbf{r}_2}\sqrt{(\mathbf{r}_2 - \mathbf{r}_1)^2 + \mathbf{L}^2}$$

$$\therefore \mathbf{R}_1\left[1-\frac{\mathbf{r}_1}{\mathbf{r}_2}\right]=\frac{\mathbf{r}_1}{\mathbf{r}_2}\sqrt{(\mathbf{r}_2-\mathbf{r}_1)^2+\mathbf{L}^2}$$

$$\mathbf{R}_1\left[\frac{\mathbf{r}_2-\mathbf{r}_1}{\mathbf{r}_2}\right]=\frac{\mathbf{r}_1}{\mathbf{r}_2}\sqrt{(\mathbf{r}_2-\mathbf{r}_1)^2+\mathbf{L}^2}$$

$$\therefore \quad \mathbf{R}_1=\frac{\mathbf{r}_1}{(\mathbf{r}_2-\mathbf{r}_1)}\sqrt{(\mathbf{r}_2-\mathbf{r}_1)^2+\mathbf{L}^2}$$

$$\mathbf{R}_2=\frac{\mathbf{r}_2\mathbf{R}_1}{\mathbf{r}_1}=\frac{\mathbf{r}_2}{(\mathbf{r}_2-\mathbf{r}_1)}\sqrt{(\mathbf{r}_2-\mathbf{r}_1)^2+\mathbf{L}^2}$$

2.3 Solving for Θ

Recall that:

$\Theta^{\mathbf{r}}=\frac{\mathbf{S}_1}{\mathbf{R}_1}=\frac{2\pi\mathbf{r}_1}{\mathbf{R}_1}$, with Θ expressed in radians.

Since $180°=\pi^{\mathbf{r}}$,

Then

$$\Theta°=\frac{180°}{\pi}\cdot\frac{2\pi\mathbf{r}_1}{\mathbf{R}_1}$$

$$=\frac{360°\mathbf{r}_1}{\mathbf{R}_1}$$

$$\therefore \Theta°=\frac{360(\mathbf{r}_2-\mathbf{r}_1)}{\sqrt{(\mathbf{r}_2-\mathbf{r}_1)^2+\mathbf{L}^2}}$$

2.4 Shroud Dimensions Expressed in Body Tube Diameters

The foregoing results provide a complete solution for the dimensional properties of a transition shroud. However, the expressions are based on body tube radius and this creates an extra calculation step, and margin for error, when one uses the relations to calculate the dimensions of a shroud. This is so because manufacturers generally express the dimensions of body tubes in terms of diameter, not radius. With a bit more algebra, we can re-express the equations in terms of body tube diameter.

To begin, we know that $\mathbf{r}_1 = \dfrac{\mathbf{d}_1}{2}$ and $\mathbf{r}_2 = \dfrac{\mathbf{d}_2}{2}$

where d_1 is the diameter of the smaller body tube and d_2 is the diameter of the larger body tube.

Substituting these variables into the solution for R_1 gives:

$$\mathbf{R}_1 = \frac{\mathbf{d}_1}{2\left[\frac{\mathbf{d}_2}{2} - \frac{\mathbf{d}_1}{2}\right]}\sqrt{\left[\frac{\mathbf{d}_2}{2} - \frac{\mathbf{d}_1}{2}\right]^2 + \mathbf{L}^2}$$

$$= \frac{\mathbf{d}_1}{2\left[\frac{\mathbf{d}_2 - \mathbf{d}_1}{2}\right]}\sqrt{\left[\frac{\mathbf{d}_2 - \mathbf{d}_1}{2}\right]^2 + \mathbf{L}^2}$$

$$= \frac{\mathbf{d}_1}{(\mathbf{d}_2 - \mathbf{d}_1)}\sqrt{\left[\frac{\mathbf{d}_2 - \mathbf{d}_1}{2}\right]^2 + \mathbf{L}^2}$$

$$= \frac{\mathbf{d}_1}{(\mathbf{d}_2 - \mathbf{d}_1)}\sqrt{\frac{(\mathbf{d}_2 - \mathbf{d}_1)^2}{4} + \mathbf{L}^2}$$

$$= \frac{\mathbf{d}_1}{(\mathbf{d}_2 - \mathbf{d}_1)}\sqrt{\frac{1}{4}\left[(\mathbf{d}_2 - \mathbf{d}_1)^2 + 4\mathbf{L}^2\right]}$$

$$= \frac{\mathbf{d}_1}{(\mathbf{d}_2 - \mathbf{d}_1)}\frac{1}{\sqrt{4}}\sqrt{(\mathbf{d}_2 - \mathbf{d}_1)^2 + 4\mathbf{L}^2}$$

$$= \frac{\mathbf{d}_1}{2(\mathbf{d}_2 - \mathbf{d}_1)}\sqrt{(\mathbf{d}_2 - \mathbf{d}_1)^2 + 4\mathbf{L}^2}$$

Solving for the other parameters gives:

$$\mathbf{R}_2 = \frac{\mathbf{r}_2\mathbf{R}_1}{\mathbf{r}_1} = \frac{\mathbf{d}_2}{2}\cdot\frac{2}{\mathbf{d}_1}\cdot\mathbf{R}_1 = \frac{\mathbf{d}_2\mathbf{R}_1}{\mathbf{d}_1}$$

$$= \frac{\mathbf{d}_2}{2(\mathbf{d}_2 - \mathbf{d}_1)}\sqrt{(\mathbf{d}_2 - \mathbf{d}_1)^2 + 4\mathbf{L}^2}$$

$$\Theta^\circ = \frac{360\,\mathbf{r}_1}{\mathbf{R}_1} = \frac{180\,\mathbf{d}_1}{\mathbf{R}_1} = \frac{360(\mathbf{d}_2 - \mathbf{d}_1)}{\sqrt{(\mathbf{d}_2 - \mathbf{d}_1)^2 + 4\mathbf{L}^2}}$$

3.0 Summary

This paper demonstrates how transition shrouds can be derived from the properties of cones and provides a set of formulae for calculating their dimensional properties.

Expressed in terms of body tube radius, the dimensions of a transition shroud are specified by:

$$\mathbf{R}_1 = \frac{\mathbf{r}_1}{(\mathbf{r}_2 - \mathbf{r}_1)}\sqrt{(\mathbf{r}_2 - \mathbf{r}_1)^2 + \mathbf{L}^2}$$

$$\mathbf{R}_2 = \frac{\mathbf{r}_2}{(\mathbf{r}_2 - \mathbf{r}_1)}\sqrt{(\mathbf{r}_2 - \mathbf{r}_1)^2 + \mathbf{L}^2}$$

$$\Theta^\circ = \frac{360(\mathbf{r}_2 - \mathbf{r}_1)}{\sqrt{(\mathbf{r}_2 - \mathbf{r}_1)^2 + \mathbf{L}^2}}$$

Expressed in terms of body tube diameter, the dimensions of a transition shroud are specified by:

$$\mathbf{R}_1 = \frac{\mathbf{d}_1}{2(\mathbf{d}_2 - \mathbf{d}_1)}\sqrt{(\mathbf{d}_2 - \mathbf{d}_1)^2 + 4\mathbf{L}^2}$$

$$\mathbf{R}_2 = \frac{\mathbf{d}_2}{2(\mathbf{d}_2 - \mathbf{d}_1)}\sqrt{(\mathbf{d}_2 - \mathbf{d}_1)^2 + 4\mathbf{L}^2}$$

$$\Theta^\circ = \frac{360(\mathbf{d}_2 - \mathbf{d}_1)}{\sqrt{(\mathbf{d}_2 - \mathbf{d}_1)^2 + 4\mathbf{L}^2}}$$